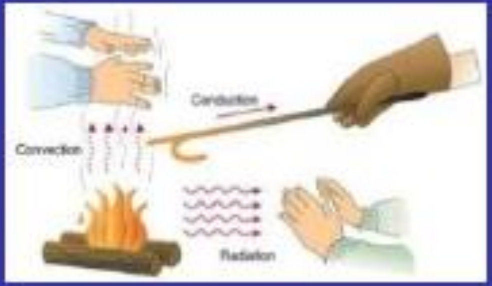
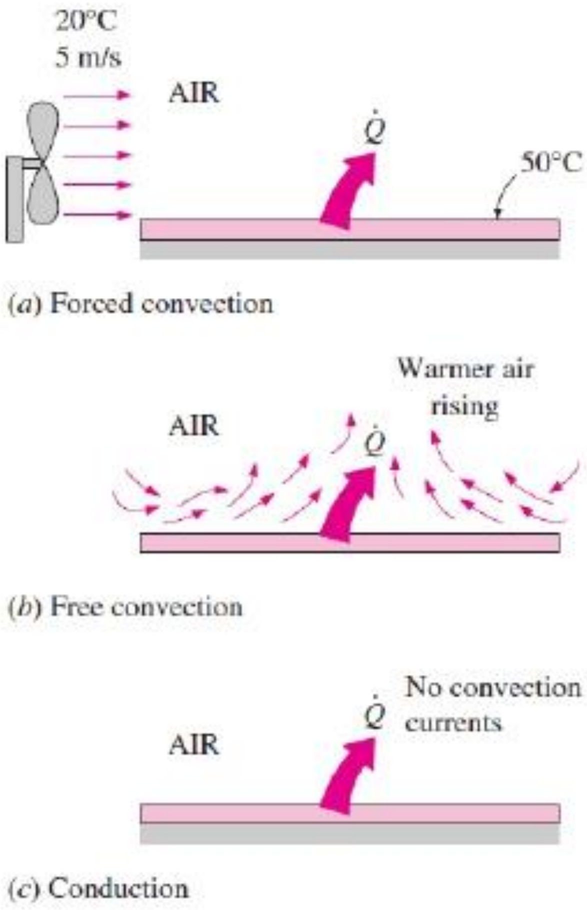
Heat and Mass Transfer



Introduction to Convection

Convection



Convective heat transfer involves

- fluid motion
- heat conduction

The fluid motion enhances the heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in fluid.
Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction.

Higher the fluid velocity, the higher the rate of heat transfer.

Convection heat transfer strongly depends on

- fluid properties: μ, k, ρ, C_p
- fluid velocity: V
- geometry and the roughness of the solid surface
- type of fluid flow (laminar or turbulent)

Newton's law of cooling

$$q_{conv} = hA_s(T_s - T_\infty)$$

T_∞ is the temp. of the fluid sufficiently far from the surface

Local heatflux

$$q''_{conv} = h_l(T_s - T_\infty)$$

h_l is the local convection coefficient

Flow conditions vary on the surface: q'', h vary along the surface.

The total heat transfer rate q:

$$q_{conv} = \int_{A_s} q'' dA_s$$

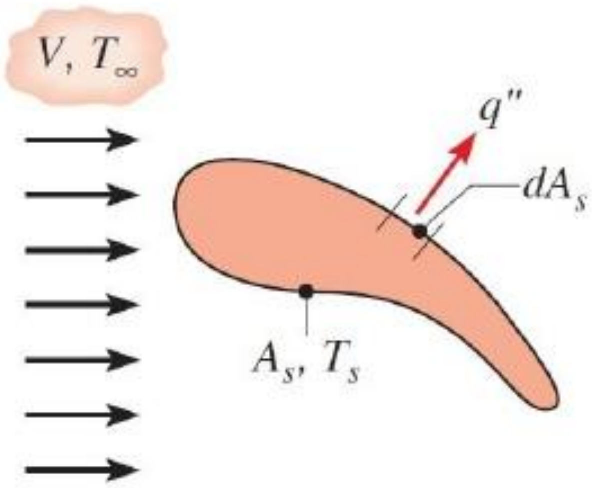
$$= (T_s - T_\infty) \int_{A_s} h dA_s$$

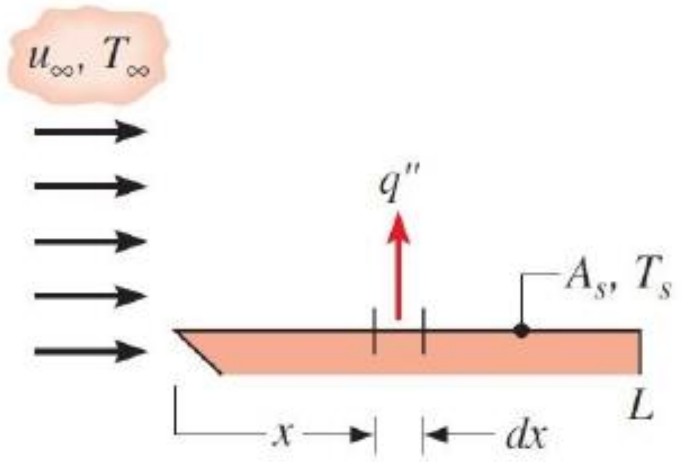
Total Heat Transfer Rate

Defining an *average convection coefficient* $\bar{h}$ for the entire surface,

$$q_{conv} = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s$$





No-Slip, No-Temperature-Jump

With no-slip and the no-temperature-jump conditions: the heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction.

$$q''_{conv} = q''_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

T represents the temperature distribution in the fluid $(\partial T/\partial y)_{y=0}$ *i.e.*, the temp. gradient at the surface.

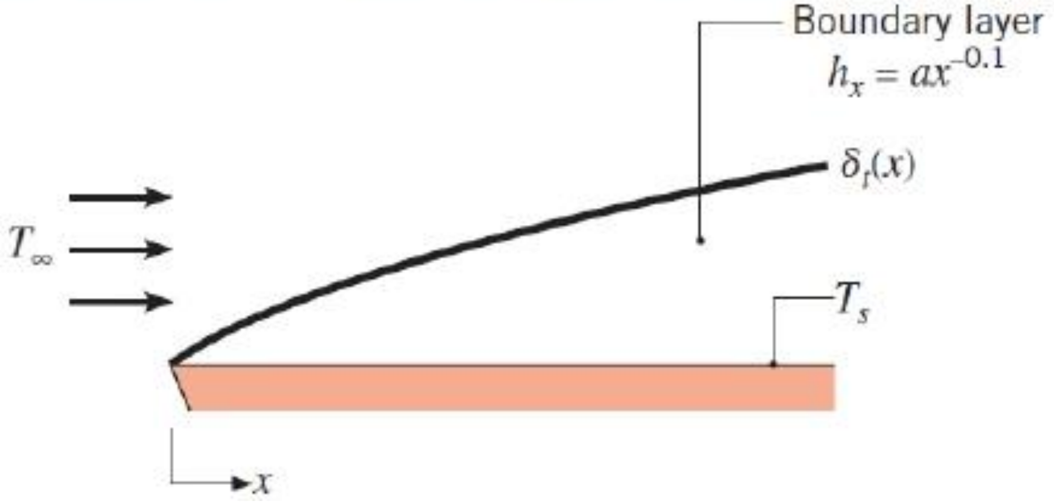
$$q''_{conv} = h(T_s - T_\infty)$$

$$h = \frac{-k_{fluid} \left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_s - T_\infty}$$

Problem

Experimental results for the local heat transfer coefficient hx for flow over a flat plate with an extremely rough surface were found to fit the relation $h_x(x) = x^{-0.1}$ where x (m) is the distance from the leading edge of the plate.

- Develop an expression for the ration of the average heat transfer coefficient $\bar{h}_x$ for a path of length x to the local heat transfer coefficient h_x at x.
- Plot the variation of h_x and $\bar{h}_x$ as a function of x.



Solution

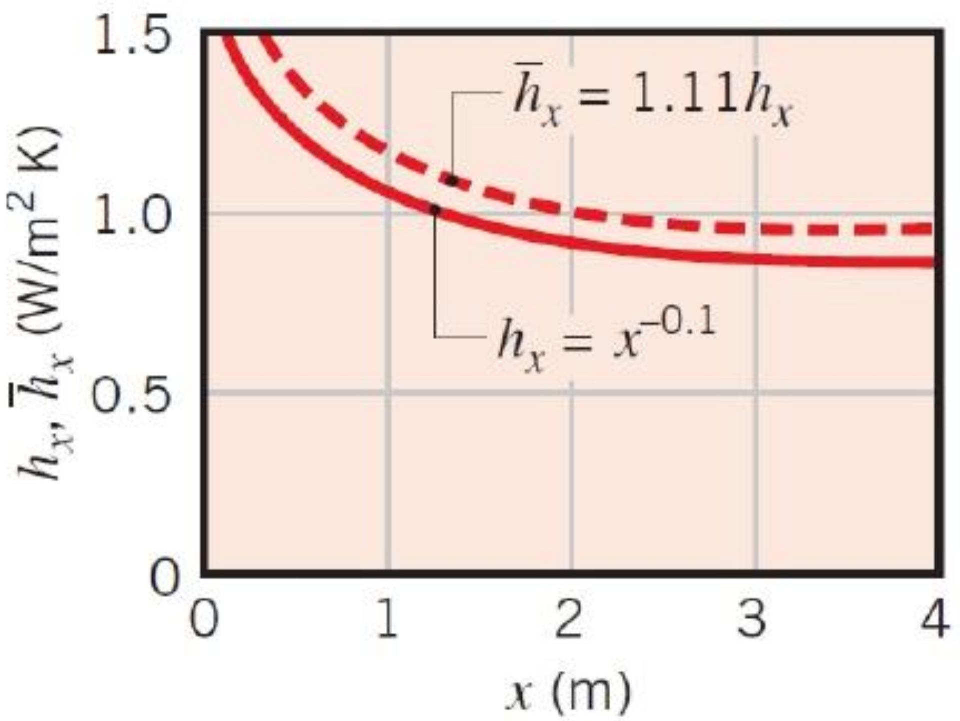
The average value of h over the region from 0 to x is:

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x(x)\,dx$$

$$= \frac{1}{x}\int_0^x x^{-0.1}\,dx$$

$$= \frac{1}{x}\frac{x^{0.9}}{0.9} = 1.11x^{-0.1}$$

$$\boxed{\bar{h}_x = 1.11h_x}$$



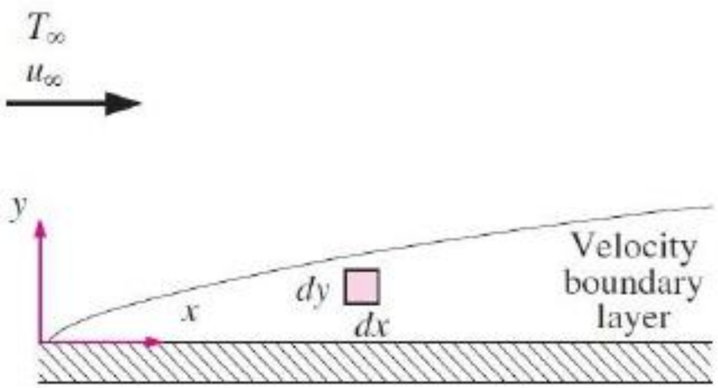
Comments

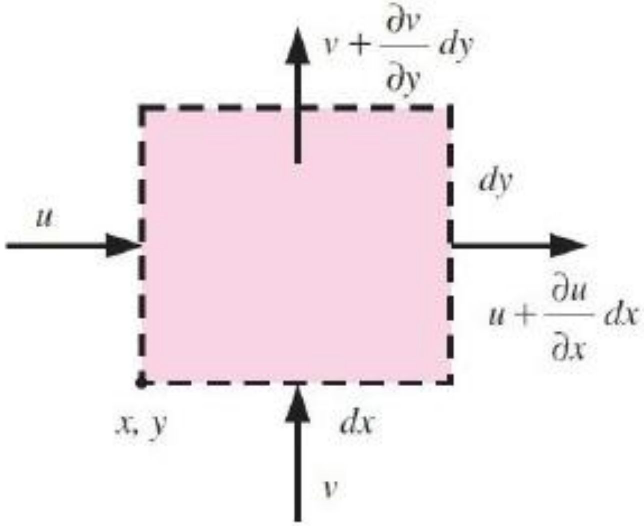
Boundary layer development causes both h_l and $\bar{h}$ to decrease with increasing distance from the leading edge. The average coefficient up to x must therefore exceed the local value at x.

Convection Equations

Assuming the flow/fluid to be:

- 2D, Steady
- Newtonian
- constant properties (ρ, μ, k, etc.)





Continuity Equation

Rate of massflow into CV = Rate of massflow out of CV

rate of fluid entering CV_{left}:

$$\rho u(dy \cdot 1)$$

rate of fluid leaving CV_{right}:

$$\rho(u + \frac{\partial u}{\partial x}dx)(dy \cdot 1)$$

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho(u + \frac{\partial u}{\partial x}dx)(dy \cdot 1) + \rho(v + \frac{\partial v}{\partial y}dy)(dx \cdot 1)$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

Momentum Equation

Expressing Newton's second law of motion for the control volume:

(Mass) (Acceleration in x) = (Net body and surface forces in x)

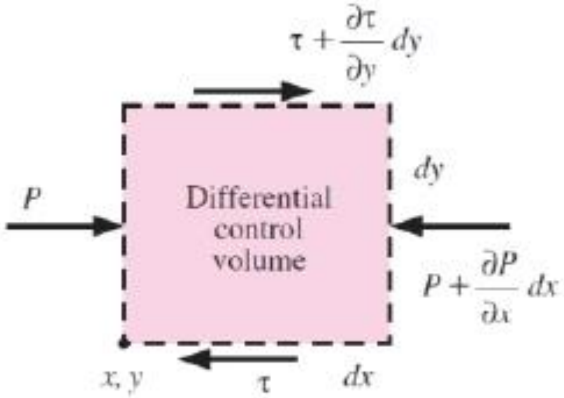
$$\delta m \cdot a_x = F_{\text{surface},x} + F_{\text{body},x} \qquad (3)$$

$$\delta m = \rho(dx \cdot dy \cdot 1)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad (4)$$

Steady state doesn't mean that acceleration is zero.
Ex: Garden hose nozzle



$$F_{\text{surface},x} = \left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1)$$

$$= \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

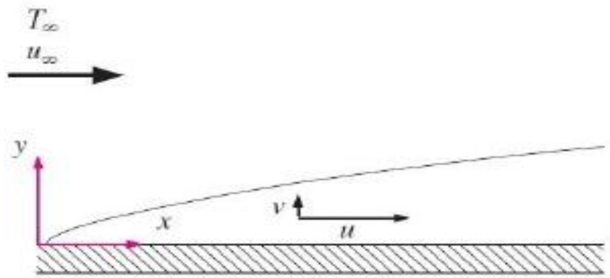
$$\tau = \mu \left(\frac{\partial u}{\partial y}\right)$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \qquad (5)$$

Combining Eqs. (3), (4) and (5):

$$\rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

x-momentum equation



1) Velocity components:
$v \ll u$

2) Velocity grandients:
$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$

3) Temperature gradients:
$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

$$\frac{\partial P}{\partial y} = 0$$

y-momentum equation

$$P = P(x) \Longrightarrow \frac{\partial P}{\partial x} = \frac{dP}{dx}$$

Energy Equation with Viscous Shear Stresses

When viscous shear stresses are not neglected, then:

$$\underbrace{\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right)}_{\text{convection}} = \underbrace{k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)}_{\text{conduction}} + \underbrace{\mu\Phi}_{\text{viscous dissipation}}$$

where the viscous dissipation term is given as:

$$\mu\Phi = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2\mu\left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right]$$

This accounts for the rate at which mechanical work is irreversibly converted to thermal energy due to viscous effects in the fluid.

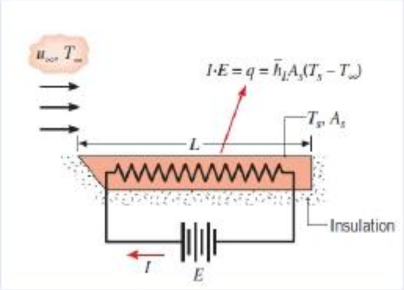
Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces
Colburn j factor (j_H)	$St\, Pr^{2/3}$	Dimensionless heat transfer coefficient
Jakob number (Ja)	$\frac{c_p(T_s - T_{sat})}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid–vapor phase change
Mach number (Ma)	$\frac{V}{a}$	Ratio of velocity to speed of sound
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Ratio of convection to pure conduction heat transfer
Peclet number (Pe_L)	$\frac{VL}{\alpha} = Re_L\, Pr$	Ratio of advection to conduction heat transfer rates
Prandtl number (Pr)	$\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities
Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces

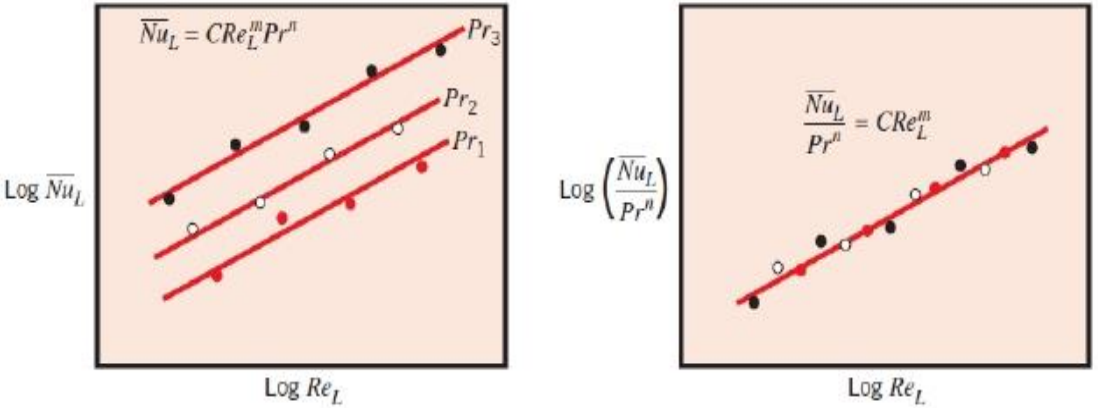
Local Nusselt number: $Nu_x = f\ (x^*, Re_L, Pr)$

Average Nusselt number: $Nu_L = f\ (Re_L, Pr)$

A common form: $Nu_L = C Re_L^m Pr^n$

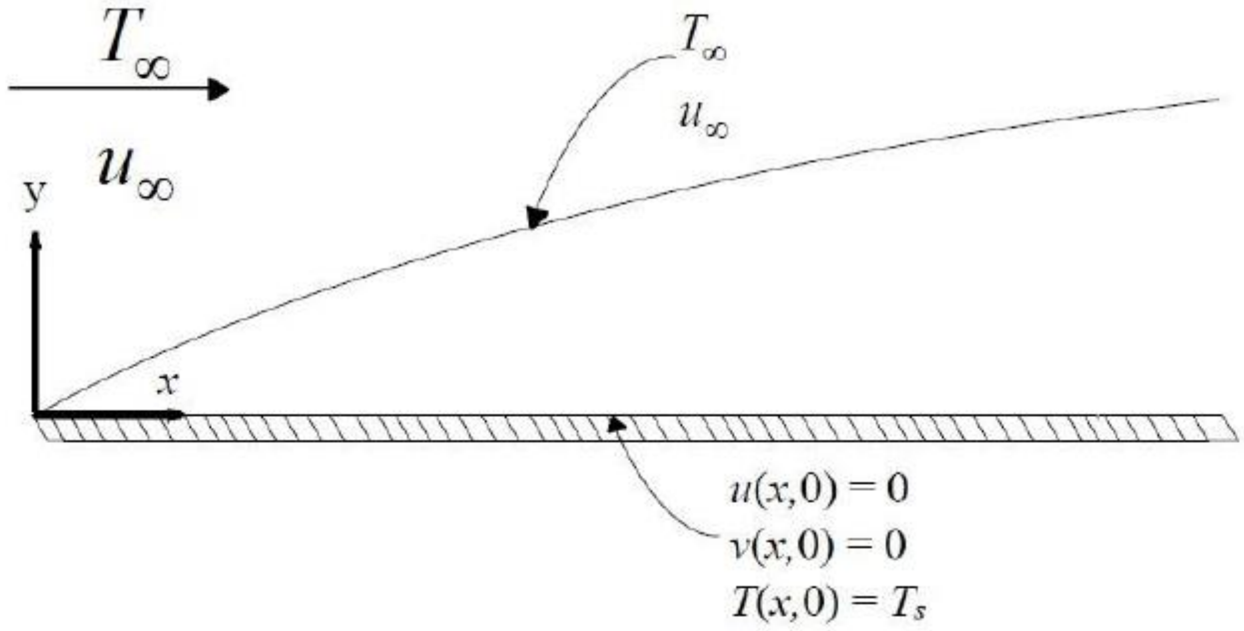
The Empirical Method





$$T_f \equiv \frac{T_s + T_\infty}{2}$$

Flat Plate in Parallel Flow



Assumptions

Steady, incompressible, laminar flow with constant fluid properties and negligible viscous dissipation.

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}$$

First solved in 1908 by German engineer H. Blasius, a student of L. Pradtl. The profile u/u_∞ remains unchanged with y/δ. A stream function $\psi(x, y)$ is defined as,

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

This takes care of continuity equation.

A dimensionless independent similarity variable and a dependent variable such that $u/u_\infty = f^{j}(\eta)$,

$$\eta = y \sqrt{\frac{u_\infty}{\nu x}} \quad \text{and} \quad f(\eta) = \frac{\psi}{u_\infty \sqrt{\nu x / u_\infty}}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = u_\infty \frac{df}{d\eta} = u_\infty f^{j}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}\left(\eta \frac{df}{d\eta} - f\right)$$

$$\left(\because -2x\frac{\partial f}{\partial x} = \eta \frac{\partial f}{\partial \eta}\right)$$

$$\boxed{2f^{jjj} + f f^{jj} = 0}$$

The problem reduced to one of solving a nonlinear third-order ordinary differential equation.

$$\boxed{2f^{jjj} + ff^{jj} = 0}$$

A third-order nonlinear differential equation with boundary conditions:

$$u(x,0) = v(x,0) = 0 \text{ and } u(x,\infty) = u_\infty$$

$$\frac{df}{d\eta}\bigg|_{\eta=0} = f(0) = 0 \text{ and } \frac{df}{d\eta}\bigg|_{\eta\to\infty} = 1$$

The problem was first solved by Blasius using a power series expansion approach, and this original solution is known as the Blasius solution.

Similarity function f and its derivatives for laminar boundary layer along a flat plate.

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

$f' = u/u_\infty = 0.99$, for $\eta = 5.0$

$$y_{\eta=5.0} = \delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5x}{\sqrt{Re_x}}$$

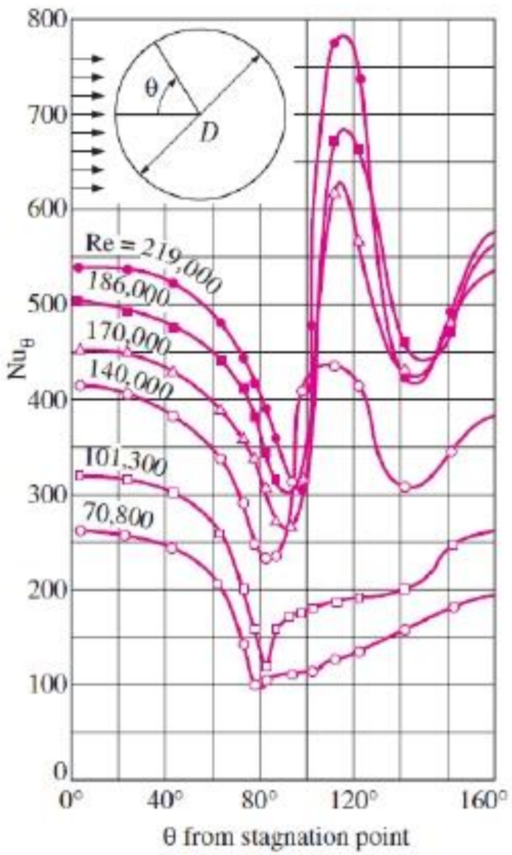
As $\delta\uparrow$ with x, $\nu\uparrow$ but $\delta\downarrow$ with $u_\infty\uparrow$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0}$$

$$\Rightarrow \tau_w = 0.332 u_\infty \sqrt{u_\infty/\nu x}$$

$$C_{f,x} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664 Re_x^{-1/2}$$

Unlike δ, τ_w and $C_{f,x}$ decrease along the plate as $x^{-1/2}$.

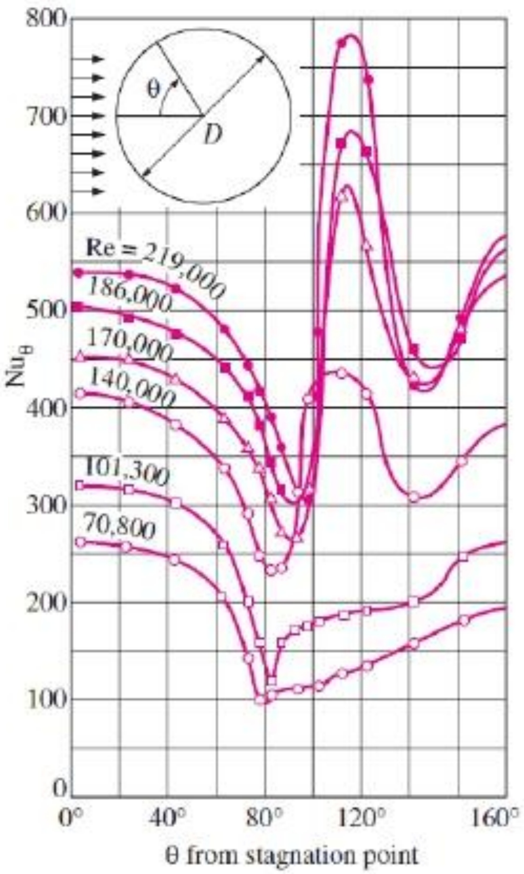


Churchill and Bernstein correlation:

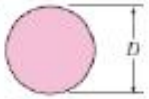
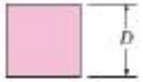
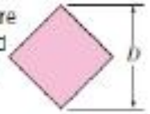
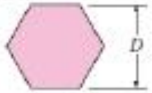
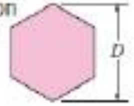
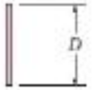
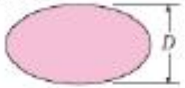
$$Nu_{cyl} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- Nu is relatively high at the stagnation point. Decreases with increasing θ as a result of the thickening of the laminar boundary layer.
- Minimum at 80°, which is the separation point in laminar flow.

Flow Across Cylinder



- Increases with increasing as a result of the intense mixing in the separated flow region (the wake).
- The sharp increase at about 90° is due to the transition from laminar to turbulent flow.
- The later decrease is again due to the thickening of the boundary layer.
- Nu reaches its second minimum at about 140°, which is the flow separation point in turbulent flow, and increases with as a result of the intense mixing in the turbulent wake region.

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

All properties are evaluated at T_f

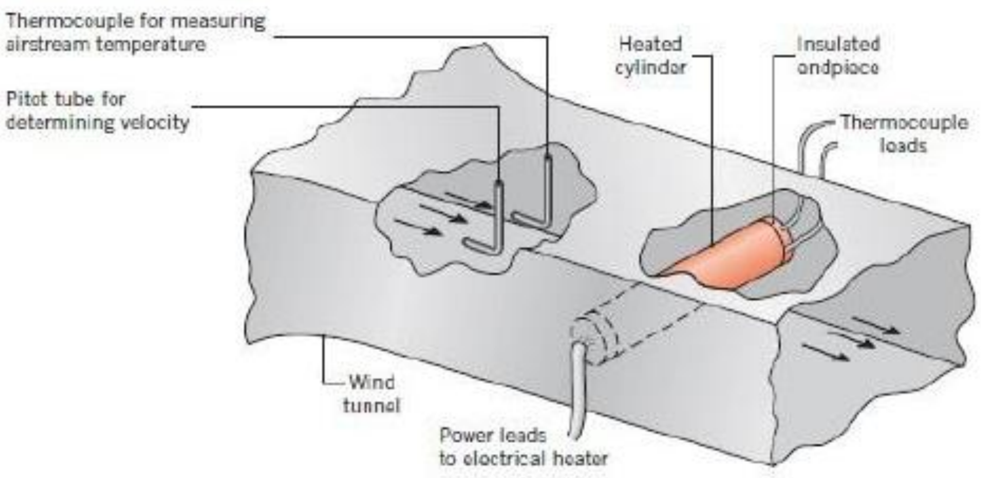
Methodology For a Convection Calculation

- Become immediately cognizant of the flow geometry.
- Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature.
- Calculate the Reynolds number.
- Decide whether a local or surface average coefficient is required.
- Select the appropriate correlation.

Problem

Experiments have been conducted on a metallic cylinder (D = 12.7 mm, L = 94 mm). The cylinder is heated internally by an electrical heater and is subjected to a cross flow of air in a low-speed wind tunnel ($V = 10$ m/s, 26.2°C). The heater power dissipation was measured to be $P = 46$ W, while $T_s = 128.4$°C. It is estimated that 15% of the power dissipation is lost through conduction and radiation.

1. Determine h from experimental observations.
2. Compare the result with appropriate correlation(s).



Solution

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$$Nu_D = 0.26\, Re_D^{0.6} Pr^{0.37} (Pr/Pr_s)^{1/4}$$ (Zhukauskasa relation)

Air ($T_\infty = 26.2$°C):
$\nu = 15.89 \times 10^{-6}$ m²/s, $k = 26.3 \times 10^{-3}$ W/m K, Pr = 0.707

Air ($T_f = 77.3$°C):
$\nu = 20.92 \times 10^{-6}$ m²/s, $k = 30 \times 10^{-3}$ W/m K, Pr = 0.700

Air ($T_s = 128.4$°C): Pr = 0.690

$$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

(Churchill relation)

Problem Sphere

The decorative plastic film on a copper sphere of 10 mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an airstream at 1 atm and 23°C having a velocity of 10 m/s. Estimate how long it will take to cool the sphere to 35°C.

Copper (T = 55°C):

$\rho = 8933$ kg/m^3, $k = 399$ W/m K, $C_p = 387$ J/kg

Air ($T_\infty = 23$°C):

$\mu = 181.6 \times 10^{-7}$ Ns/m^2, $\nu = 15.36 \times 10^{-6}$ m^2/s,

$k = 0.0258$ W/m K, Pr = 0.709

Air ($T_s = 55$°C): $\mu = 197.8 \times 10^{-7}$ Ns/m^2

$$Nu_D = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

All properties except μ_s are evaluated at T_∞.

The decorative plastic film on a copper sphere of 10 mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an airstream at 1 atm and 23°C having a velocity of 10 m/s. Estimate how long it will take to cool the sphere to 35°C.

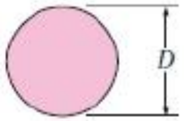
Copper ($T = 55°C$):

$\rho = 8933$ kg/m^3, $k = 399$ W/m K, $C_p = 387$ J/kg

Air ($T_\infty = 39°C$):

$\nu = 17.15 \times 10^{-6}$ m^2/s, Pr = 0.705

Air ($T_s = 55°C$): $\mu = 197.8 \times 10^{-7}$ Ns/m^2

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle  D	Gas or liquid	0.4–4 4–40 40 4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330} Pr^{1/3}$ $Nu = 0.911Re^{0.385} Pr^{1/3}$ $Nu - 0.683Re^{0.466} Pr^{1/3}$ $Nu = 0.193Re^{0.618} Pr^{1/3}$ $Nu = 0.027Re^{0.805} Pr^{1/3}$

All properties are evaluated at T_f

Internal Flow

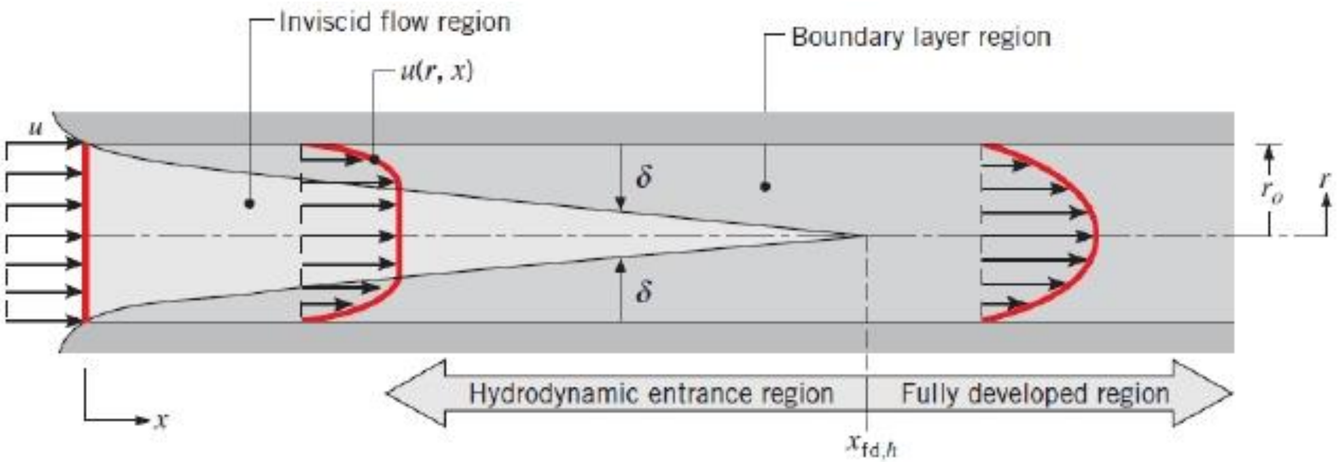
External flow

- Fluid has a free surface
- δ is free to grow indefinitely

Internal flow

- Fluid is completely confined by the inner surfaces of the tube
- There is a limit on how much δ can grow

Circular pipes can withstand large pressure difference between inside and outside without distortion. Provide the most heat transfer for the least pressure drop.

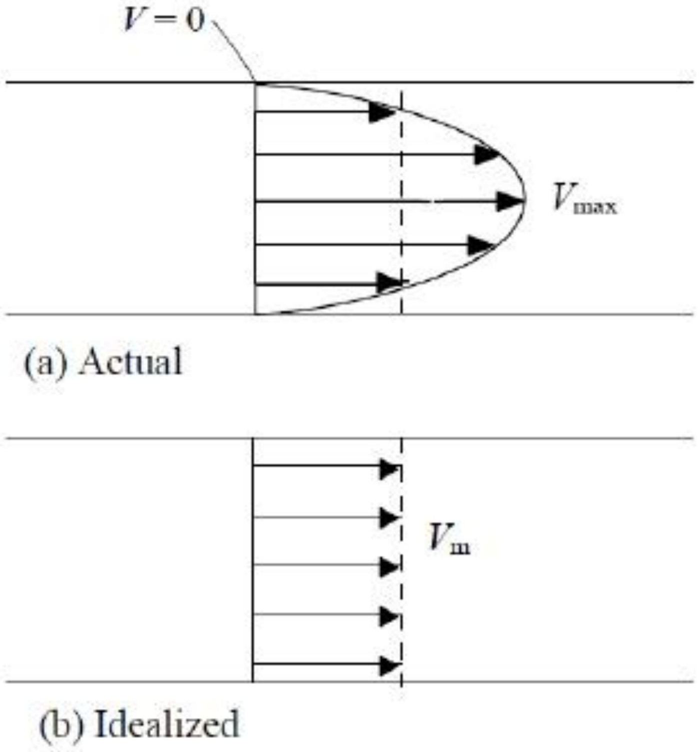


$$Re_D = \frac{\rho u_m D}{\mu}$$

Critical $Re_{D,c} \approx 2300$

Laminar $\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05 Re_D$

Turbulence $\left(\frac{x_{fd,h}}{D}\right)_{turb} \approx 10$

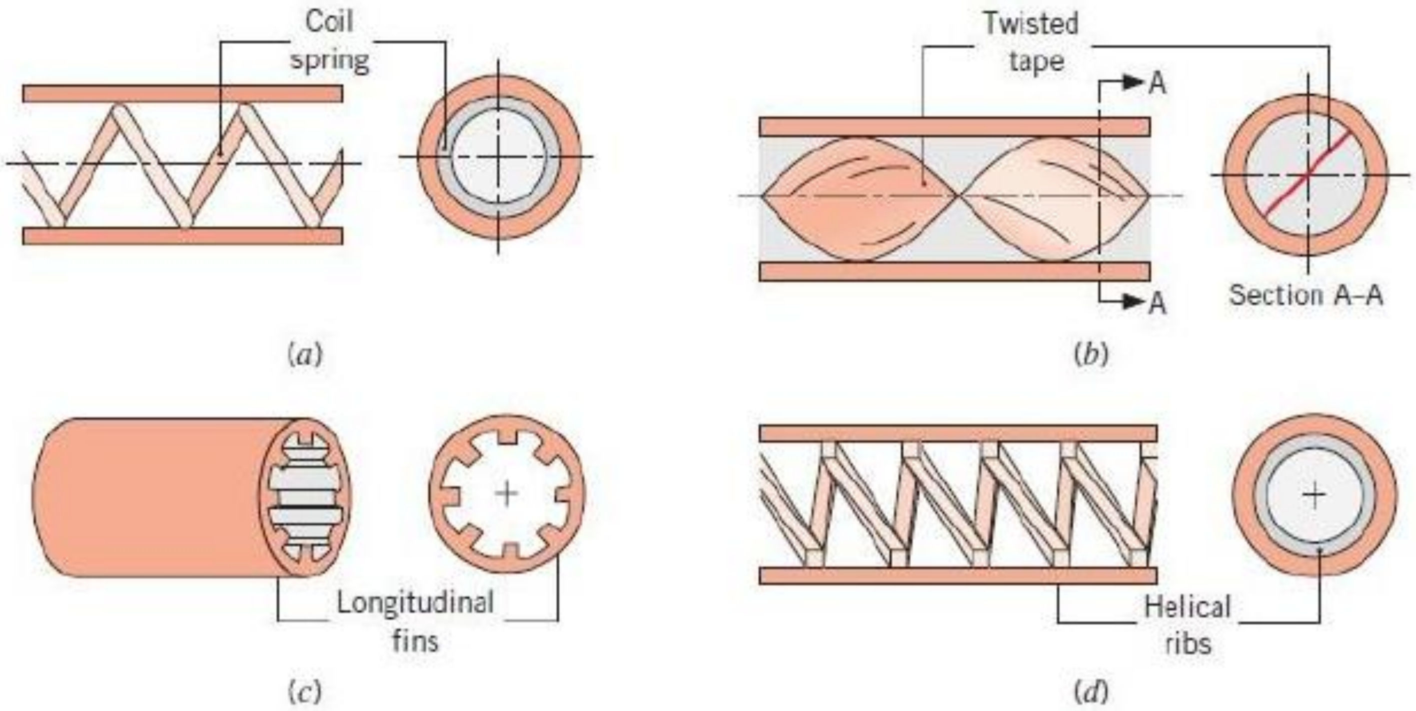


$$\dot{m} = \rho u_m A_c = \int_{A_c} \rho u(r,x)\, dA_c$$

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r,x)\, r\, dr$$

Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D = \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	$f Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
a, b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53



Internal flow heat transfer enhancement schemes: (*a*) longitudinal section and end view of coil-spring wire insert, (*b*) longitudinal section and cross-sectional view of twisted tape insert, (*c*) cut-away section and end view of longitudinal fins, and (*d*) longitudinal section and end view of helical ribs.